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LIQUIDBORNE

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PREPARED FOR BUREAU OF SHIPS

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Task 1359
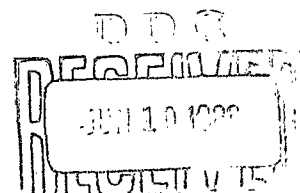
ENGINEERING

RESEARCH

REPORT

SCHOOL OF CIVIL ENGINEERING
OKLAHOMA STATE UNIVERSITY

SCHOOL OF CIVIL ENGINEERING

Research Publication No. 8

Oklahoma State University

Stillwater, Oklahoma

LIQUIDBORNE NOISE REDUCTION

Volume II

FINAL REPORT OF CONTRACT NObs 86437

By

EDWIN J. WALLER

and

LARRY E. HOVE

February, 1963

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February, 1963
REFERENCE: Contract NObs. 86437
SF-013-11-01
Task 1359

TO: Chief, Bureau of Ships
            Department of the Navy
            Washington 25, D. C.
            ATTN: Code 345

FROM: Edwin J. Waller, Project Director
            School of Civil Engineering
            Oklahoma State University
            Stillwater, Oklahoma

SUBJECT: Final Report

This report summarizes the work performed under contract

NObs 86437 during the period February 1, 1962, to January 31, 1963.

Work is continuing under contract NObs 88297 which terminates Jan-

uary 31, 1964.

The over-all objective of this project was to aid the Bureau

of Ships in prevention of excessive liquidborne noise levels at critical

points in shipboard piping systems. This entails the following:

A. To develop a standard method of evaluating the noise
producing characteristics of pumps and flow control devices
in piping systems.

B. To develop the necessary information leading to the pre-
paration of standards and specifications for pumps and
control valves by studying the response of various typical
piping arrangements to noise inputs typical of the pumps
and control devices and presenting the results in terms
of the acoustical levels at the discharge of the piping sys-
tem.

C. To carry out necessary mathematical analyses as a part
of B. above which yield pertinent information useful for
improving design and analysis techniques for the over-all
pump-piping system including design of noise filters.

Liquidborne noise is generated in pumps and control valves by

1.

fluid turbulence, cavitation, and by the acceleration of the liquid by moving mechanical parts such as vanes or pistons. The noise thus created at the pump or valve is transmitted (i.e. propagated) in the form of pressure pulsations throughout the piping system. Additional noise is generated by the fluid as it flows through the piping system, depending upon its configuration. This project was concerned only with the noise which is generated at the pump or control valve.

The noise intensity at any given point in the system depends on the dynamic response capability of the piping system and the intensity of the noise generated by the pump or control device. There are two fundamental means of altering the noise level. These are either to redesign the piping system, minimizing the noise, or to decrease the noise level at the source.

Generally the piping system and the pumps are designed by different persons for performance criteria which gives little or no consideration of noise generation and subsequent transmission. In some systems the noise level is acceptable while in others it is not. This is due to differences in the piping system parameters such as elasticity, viscosity, and density of the liquid; the elasticity, diameter, and length of the pipe; and the terminating conditions of the piping system. Thus, the parameters of the piping system have considerable influence on the noise spectrum.

Figure 1a shows the types of discontinuities encountered in shipboard piping systems. These include changes in size (A), looping of lines (B to C), and stub lines (D). Additional complications arise when multiple sources and multiple terminal points are looped into one system, as indicated by Figure 1b.
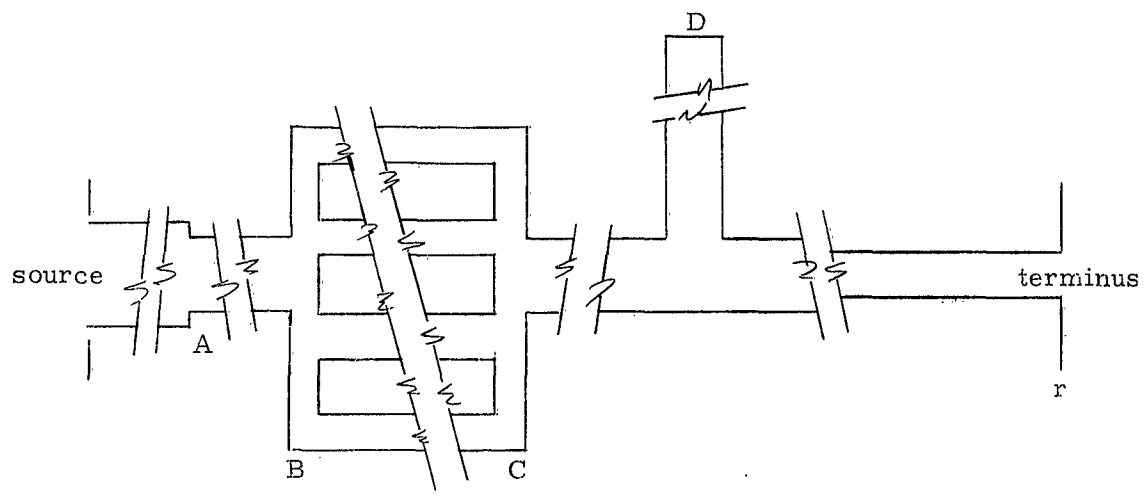
Figure 1a.
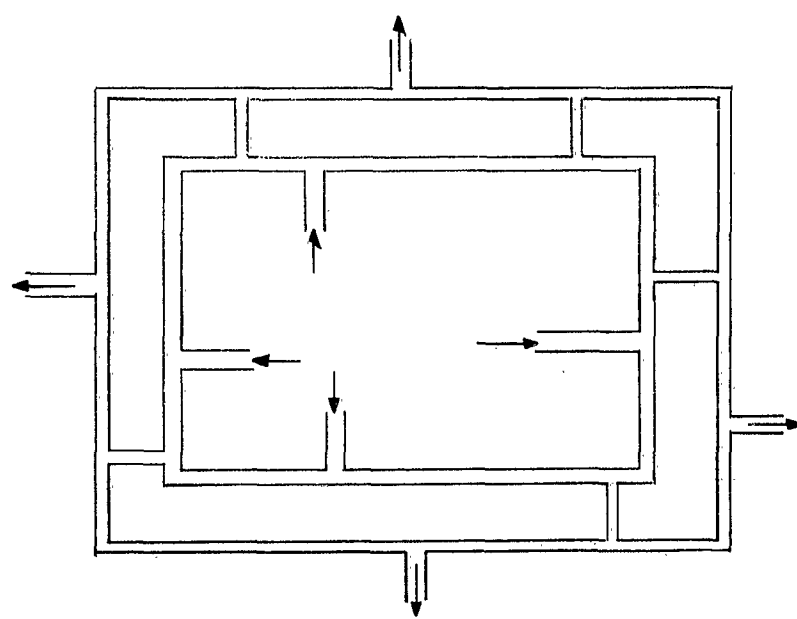
Figure 1b.

Specific Objectives and Resume of Results.

A literature survey of pertinent publications and reports was the first objective of this project. Material was supplied by the Oklahoma State University Library and from the Bureau of Ships. The results are shown in the Bibliography of this report. Although the results of the survey were informative, no practical method of determining the response of complex piping systems was found in the literature.

The next objective was to establish a practical method of determining the response of the complex piping system by mathematical analysis. This analysis was carried out and presented with The Third Quarterly Report of this project. Necessary revisions have been made and are presented in Chapters I to IV of this report. The results include the mathematical methods used in the analysis, the idealizations involved in the physical model of the piping system, the techniques and formulas for finding the transfer functions relating the pressure spectra at various points in the system, and general problems involving input specification. The results are formidable, but indicate a technique of analysis which can be carried out. The limitation is computer facilities.

Considerable time was spent in developing and evaluating various analysis methods with the objective of finding the most feasible procedure. The most promising of these is discussed in Chapter V. Although the transfer functions developed in Chapter III are valid for all frequencies, a substantial amount of information could be gained from the maximum and minimum of the transfer functions if they could be determined. However, the results shown in Chapters V and VI

giving upper and lower bounds of pressure spectra are unreasonably extreme and impractical for use in a simplified analysis.

Other results are shown in Chapter VI. These include transfer function calculations for suction lines in a system similar to the auxiliary sea water system of the USS TINOSA (SS(N)606) in which allowable pressure spectra are found for the frequency range $0 < f < 10,000$ cps. The results "de-bugged" part of a general program which will be used in future studies. Essentially this technique involved calculation of the transfer functions in complex number form.

The mathematical technique under development at Oklahoma State University is being adapted for computer solution. The programs developed will use realistic piping and flow data obtained from the auxiliary sea water system of the TINOSA. During the last two months of the contract period, some time was spent in collecting information such as attenuation, characteristic impedance, and phase constants for the sea water system from the drawings of the system, and in analysis of heat exchangers to determine their dynamic response. Work continues toward the over-all goals.

CHAPTER I

TRANSFORM METHODS IN SYSTEM ANALYSIS

1.1 Introduction.

Operational calculus, or transform methods, has proven to be an elegant tool in the analysis of linear systems. The system under consideration is no exception. Certain relationships involving transform methods are dealt with in this chapter.

1.2 Fourier Series.

Adequate description of the dependent variable(s) involved in an analysis is important. For periodic functions, this is often accomplished by the Fourier series or, as to be used here, the complex form of the Fourier series.

Although any independent variable can be used, the dependent variable will be considered a function of time f(t) and periodic in time T. Then [108]*,

$$f(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(\omega_n) e^{j\omega_n t} \ , \quad j = \sqrt{-1} \tag{1.1}$$

where the complex coefficient

$$F(\omega_n) = \int_{-T/2}^{T/2} f(t) e^{-j\omega_n t} dt \ , \tag{1.2}$$

and the nth harmonic of the fundamental angular frequency

---

* Numbers in parentheses refer to references in Bibliography.

$$\omega_n = \frac{2\pi n}{T} \ .$$

(1.3)

1.3 Fourier and Laplace Transforms.

Mathematical definitions and justifications of Fourier and Laplace transforms are available in many sources [99, 114]. The purpose of this section is to establish the interrelationship of the Fourier series description of a function and the Fourier transform of the function [108].

Equation (1.1) can be written

$$f(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} F(\omega_n) e^{j\omega_n t} \omega_1$$

(1.4)

where ω_1 is the fundamental angular frequency. As the period T tends to infinity, the function f(t) tends to be aperiodic, the fundamental angular frequency ω_1 becomes a differential of angular frequency $d\omega$, and the nth harmonic angular frequency ω_n becomes a continuous angular frequency ω. In the limit, f(t) becomes aperiodic, and Equation (1.4) becomes

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

(1.5)

where

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

(1.6)

from Equation (1.2).

Equations (1.5) and (1.6) are recognized as a Fourier transform pair. The corresponding Laplace transform relationships are

$$f(t) = \frac{1}{2\pi j} \int_{\epsilon-j\infty}^{\epsilon+j\infty} F(s) e^{st} ds,$$

(1.7)

and

$$F(s) = \int_0^\infty f(t) e^{-st} dt \qquad (1.8)$$

where s is the complex variable $\sigma + j\omega$. If the function f(t) is defined as

$$f(t) = \begin{cases} f(t), & t \geq 0 \\ 0, & t < 0, \end{cases} \qquad (1.9)$$

it is evident that the Laplace and Fourier transform pairs become identical when s is replaced by $j\omega$.

1.4 Spectrum Analysis [108].

From the results of preceding discussion, it is concluded that a dependent variable can be described in either the time or frequency domain. In the time domain, the variable can be expressed by an appropriate analytical function or by a Fourier series if periodic.

The frequency domain description of a periodic function is evident from Equation (1.1). The function

$$\frac{1}{T} F(\omega_n)$$

for $n = 0, \pm 1, 2, 3 \ldots$ gives a measure of the periodic component of f(t) at the nth harmonic frequency ω_n. In general, $F(\omega_n)$ is complex, and

$$\frac{1}{T} F(\omega_n) = \frac{1}{T} \left| F(\omega_n) \right| e^{j\theta_n} \qquad (1.10)$$

where θ_n is the phase angle of $F(\omega_n)$. Plotting

$$\frac{1}{T} \left| F(\omega_n) \right|$$

versus ω_n (or n) results in a discrete spectrum known as the amplitude spectrum of f(t). Similarly, the graph θ_n versus ω_n (or n) is the phase spectrum.

For the non-periodic function f(t), the function

$$\frac{1}{2\pi} \; F(\omega) = \frac{1}{2\pi} \; \left| \; F(\omega) \; \right| \; e^{j\theta(\omega)} \tag{1.11}$$

is a continuous function of ω and is in general complex. It is the complex continuous spectrum of f(t). The amplitude density spectrum of f(t) is

$$\frac{1}{2\pi} \; \left| \; F(\omega) \; \right| \; ,$$

and its phase density spectrum is $\theta(\omega)$. Since division by 2π in Equation (1.11) is simply a normalizing factor, the function $\left| F(\omega) \right|$ shall hereafter be referred to as the amplitude density spectrum of f(t).

1.5 Transfer Functions and Frequency Response.

The analysis of a linear system by Laplace transform methods often results in a relationship of the form [99, 114]

$$F_o(s) = H(s) \; F_i(s) \tag{1.12}$$

where $F_o(s)$ and $F_i(s)$ are the Laplace transforms of the system output $f_o(t)$ and input $f_i(t)$, respectively. The function H(s) is usually referred to as a transfer function or system function.

If $f_i(t)$ is defined as in Equation (1.9), the variable s can be replaced by $j\omega$ in Equation (1.12) to give [*]

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*When s is replaced by $j\omega$ in a function F(s), the result will be denoted $F(\omega)$ rather than $F(j\omega)$.

$$F_o(\omega) = H(\omega) F_i(\omega). \tag{1.13}$$

The function $F_o(\omega)$ will be called the frequency response of the system.

The amplitude density spectrum of the system output in terms of the input spectrum and system function follows by taking the absolute value of both sides of Equation (1.13). Thus,

$$|F_o(\omega)| = |H(\omega)| |F_i(\omega)| . \tag{1.14}$$

The term transfer function will be used interchangeably to describe either $H(s)$, $H(\omega)$, or $|H(\omega)|$.

CHAPTER II

PHYSICAL SYSTEM

2.1 Introduction.

This chapter contains the following:

1. Derivation of differential equations describing compressible, turbulent flow of water or similar liquid in a non-rigid cylindrical conduit.

2. Discussion of conditions imposed on system.

3. Laplace transform domain solution of describing equations.

4. System frequency response and discussion of system parameters.

The results of this investigation comprise a major part of the necessary information required to determine the response of a piping system such as the one shown in Figure 1a.

2.2 Derivation of Describing Equations.

Figure 2 depicts a liquid-filled cylinder with non-rigid walls under compressible, turbulent flow conditions. Conditions imposed on the system follow and hereafter will be referred to by number:

1. The flow is one-dimensional.

2. The frictional resistance is a function of some nonlinear operation on the fluid velocity and system parameters. It can be expressed as the product of a constant and the instantaneous flow rate variation.

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3. All energy dissipation is accounted for by Condition 2.

4. The rate of change in fluid velocity with respect to length is negligible as compared to its rate of change with respect to time.

5. The instantaneous pressure is constant over the cross section.

6. The rate of change in mass density with respect to length is negligible as compared to its rate of change with respect to time.

7. The velocity of wave propagation in the liquid cylinder is constant.
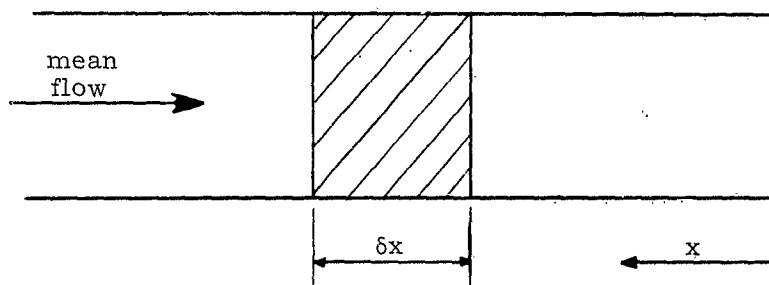
Figure 2.

The total velocity u_t of a liquid particle is considered to be the sum of a mean velocity $\bar{u}$ and an instantaneous velocity variation u assumed positive in the direction of mean flow, i.e., $u_t = \bar{u} + u$. Likewise, the total pressure p_t is the sum of a mean pressure $\bar{p}$ and an instantaneous pressure variation p, i.e., $p_t = \bar{p} + p$.

Considering conditions 1, 2, 3, 4, and 6 and assuming the positive lateral direction x to oppose the mean flow, the following relationships will completely describe the system [112].

Equation of motion:

$$\frac{\partial u_t}{\partial t} - \frac{1}{\rho} \frac{\partial p_t}{\partial x} + Gu_t^n = 0 \tag{2.1}$$

where ρ is the liquid mass density and G is a constant.

Equation of continuity:

$$-\rho \frac{\partial u_t}{\partial x} + \frac{\partial \rho}{\partial t} = 0 \tag{2.2}$$

Equation of state:

$$\partial p_t = \frac{K'}{\rho} \partial \rho \tag{2.3}$$

where K' denotes the bulk modulus of the liquid and conduit combined. It is given by [114]

$$K' = \frac{KbE}{KD + bE} \tag{2.4}$$

where K denotes liquid bulk modulus, b is conduit wall thickness, E is the modulus of elasticity of wall material, and D denotes inside diameter.

Equations (2.2) and (2.3) are combined to give

$$- \frac{\partial u_t}{\partial x} + \frac{1}{K'} \frac{\partial p_t}{\partial t} = 0 . \tag{2.5}$$

Integrating Equations (2.1) and (2.5) over a control volume $A\delta x$ and using condition 5,

$$- \frac{\partial p_t}{\partial x} + \frac{\rho}{A} \frac{\partial q_t}{\partial t} + Bq_t^n = 0 , \tag{2.6}$$

and

$$- \frac{\partial q_t}{\partial x} + \frac{A}{K'} \frac{\partial p_t}{\partial t} = 0 \tag{2.7}$$

where A denotes pipe area and B is a constant. The total volume flow rate q_t is the sum of a mean flow rate $\bar{q}$ and an instantaneous flow rate variation q assumed positive in the direction of mean flow, i.e., $q_t = \bar{q} + q$. The nonlinear equation (2.6) is linearized as follows[93].

If the flow rate is considered constant at a cross-section, Equation (2.6) becomes the slope of the pressure grade line, i.e.,

$$-\frac{\partial \bar{p}}{\partial x} = B\bar{q}^n = \frac{f\rho\bar{q}^2}{2DA^2} \tag{2.8}$$

where f is the Darcy-Weisbach friction factor. Then

$$B = \frac{f\rho\bar{q}^{2-n}}{2DA^2} = \frac{\bar{p}_f}{\ell\bar{q}^n} \tag{2.9}$$

where $\bar{p}_f$ denotes the pressure needed to overcome the frictional resistance of the pipe. The value of n is estimated from experience or experiment and should range between 1.65 and 2.05.

The nonlinear term in Equation (2.6) is rewritten and expanded as follows:

$$\begin{aligned}
Bq_t^n &= B(\bar{q} + q)^n \\
&= B\bar{q}^n (1 + \frac{q}{\bar{q}})^n \\
&= \frac{\bar{p}_f}{\ell}\left[1 + n(\frac{q}{\bar{q}}) + \frac{n(n-1)}{2!}(\frac{q}{\bar{q}})^2 + \ldots\right].
\end{aligned} \tag{2.10}$$

The series in Equation (2.10) is convergent for $q \leq \bar{q}$, and since $q \ll \bar{q}$, the series is assumed sufficiently approximated by its first two terms. Equation (2.6) is now

$$-\frac{\partial p_t}{\partial x} + \frac{\rho}{A}\frac{\partial q_t}{\partial t} + \frac{\bar{p}_f}{\ell} + \frac{n\bar{p}_f}{\ell\bar{q}}q = 0. \tag{2.11}$$

The desired describing differential equations for instantaneous variations in flow rate and pressure follow directly from Equations (2.11) and (2.7) by replacing q_t and p_t by their respective sums. They are:

$$- \frac{\partial p}{\partial x} + \frac{\rho}{A} \frac{\partial q}{\partial t} + \frac{n \bar{p}_f}{\ell \bar{q}} q = 0 ; \qquad (2.12)$$

and

$$- \frac{\partial q}{\partial x} + \frac{A}{K'} \frac{\partial p}{\partial t} = 0 . \qquad (2.13)$$

2.3 Imposed Conditions.

The conditions imposed on the physical system are discussed and validated to varying degrees in the following.

1. In any real piping system, a propagated wave is never one-dimensional, because the elasticity of the pipe walls allows radial motion. But over a certain frequency range from 0 to ω_c radians per second there exists only one propagated wave, or one mode of propagation, which is the axial, or plane wave, mode. This mode is denoted the $(0, 0)$ mode. The cut-off frequency ω_c at which the first radial mode of propagation occurs [the $(0, 1)$ mode] for nondissipative conditions can be determined by the following equations given by Jacobi [41]:

$$\frac{J_0(z)}{z J_1(z)} = \frac{2 \rho_2 b}{\rho_1 D} - \frac{2 E b}{\rho_1 D c^2 z^2} ; \qquad (2.14)$$

and

$$\omega_c = \frac{2c}{D} z_{01} \qquad (2.15)$$

where z is some characteristic value, J_0 and J_1 are Bessel functions of the first kind, and ρ_1 and ρ_2 are the mass densities of the fluid and pipe material, respectively. The first real characteristic value z_{01} that satisfies Equation (2.14) is substituted into Equation (2.15) to obtain ω_c. For 8" standard steel pipe, f_c = 6900 cycles per second. For smaller sizes of pipe the value of ω_c increases substantially. To the best knowledge of the authors, the subject of cut-off frequencies for dissipative conditions is nonexistent in the literature. The cut-off frequency should be somewhat higher for dissipative conditions than that given by Equations (2.14) and (2.15). It is concluded that for a relatively rigid pipe, the one-dimensional condition is valid for frequencies up to the cut-off frequency.

2. The perturbation process used in the derivation of the describing equation to linearize the frictional dissipation term was proposed and verified experimentally for low frequencies by Waller [93]. There seems to be no good reason for discounting its validity in the frequency range of one-dimensional propagation.

3. The effect of heat transfer on a propagated wave could be considered resulting in a third describing differential equation as done by Brown [15], but it is believed that the rather empirical frictional dissipation term used in the describing equations will account for all energy dissipation.

4. The condition that the rate of change in fluid velocity with respect to length is negligible has been used for many years

in describing wave propagation in liquid cylinders. Phillips[68], in showing the acoustic wave equation adequate for describing turbulent flow in liquid-filled steel pipes,has shown this term to be negligible.

5. If the one-dimensional wave condition is valid, it follows that the instantaneous pressure is essentially uniform over the cross section.

6. As in Condition 4, the rate of change in mass density with respect to length has classically been considered negligible.

7. Strictly speaking, the velocity of wave propagation in a liquid cylinder with non-rigid walls is not constant with frequency. With increasing frequency, the flexural vibrations of the walls tend to impede the propagation causing a decrease in the wave velocity. Mathematical expressions to determine the velocity of wave propagation as a function of frequency are given in the paper by Jacobi[41]. For a relatively rigid pipe, such as a standard steel pipe, the velocity of wave propagation in the pipe, a, is essentially constant, and[114]

$$a = \sqrt{\frac{K'}{\rho}} \ .$$ (2.16)

Rather than calculating the velocity of propagation by Equations (2.4) and (2.16), it can be determined for different pipe sizes and fluids from nomographs available in many papers and books [111].

2.4 Solution of Differential Equations.

The discussion is facilitated by introducing three system parameters. They are:

Coefficient of inertia,

$$L = \frac{\rho}{A} \; ; \tag{2.17}$$

Coefficient of resistance,

$$R = \frac{n \, \bar{p}_f}{\ell \, \bar{q}} \; ; \tag{2.18}$$

Coefficient of capacitance,

$$C = \frac{A}{K'} = \frac{1}{La^2} \; . \tag{2.19}$$

In terms of these parameters, the describing differential Equations (2.12) and (2.13) are:

$$- \frac{\partial p \, (x, t)}{\partial x} + L \frac{\partial q \, (x, t)}{\partial t} + R \, q(x, t) = 0 \; ; \tag{2.20}$$

and

$$- \frac{\partial q \, (x, t)}{\partial x} + C \frac{\partial p \, (x, t)}{\partial t} = 0 \; . \tag{2.21}$$

Using Laplace transform methods, Equations (2.20) and (2.21) are solved for pressure and flow rate in the transformed time domain as follows [96].

Transforming Equations (2.20) and (2.21) with the Laplace integral

$$F(s) = \int_0^\infty f(t) \, e^{-st} \, dt \; ,$$

$$- \frac{\partial P \, (x, s)}{\partial x} + (R + sL) \, Q(x, s) = 0 \; , \tag{2.22}$$

and

$$- \frac{\partial Q \, (x, s)}{\partial x} + s \, CP(x, s) = 0 \tag{2.23}$$

where p(x, 0) and q(x, 0) are zero. Transforming the above equations with the Laplace integral

$$\hat{F}(\lambda) = \int_0^\infty f(x) e^{-\lambda x} \, dx \ ,$$

$$-\lambda \hat{P}(\lambda, s) + (R + sL) \hat{Q}(\lambda, s) = -P(0, s) \ , \tag{2.24}$$

and

$$-\lambda \hat{Q}(\lambda, s) + sC \hat{P}(\lambda, s) = -Q(0, s) \ . \tag{2.25}$$

Solving Equations (2.24) and (2.25) for $\hat{P}(\lambda, s)$ and $\hat{Q}(\lambda, s)$,

$$\hat{P}(\lambda, s) = Z_c Q(0, s) \left(\frac{\gamma}{\lambda^2 - \gamma^2} \right) + P(0, s) \left(\frac{\lambda}{\lambda^2 - \gamma^2} \right), \tag{2.26}$$

and

$$\hat{Q}(\lambda, s) = \frac{P(0, s)}{Z_c} \left(\frac{\gamma}{\lambda^2 - \gamma^2} \right) + Q(0, s) \left(\frac{\lambda}{\lambda^2 - \gamma^2} \right) \tag{2.27}$$

where

$$\gamma^2 = sC (R + sL) \ , \tag{2.28}$$

and

$$Z_c^2 = \frac{R + sL}{sC} \ . \tag{2.29}$$

Evaluating Equations (2.26) and (2.27) with the inversion integral

$$f(x) = \frac{1}{2\pi j} \int_{\epsilon - j\infty}^{\epsilon = j\infty} \hat{F}(\lambda) e^{\lambda t} \, d\lambda \ ,$$

$$P(x, s) = P(0, s) \cosh \gamma x + Z_c Q(0, s) \sinh \gamma x \ , \tag{2.30}$$

and

$$Q(x, s) = Q(0, s) \cosh \gamma x + \frac{P(0, s)}{Z_c} \sinh \gamma x \ . \tag{2.31}$$

Equations (2.30) and (2.31) provide the Laplace transform domain solution for pressure and flow rate fluctuation at a point in a pipe in terms of a propagation coefficient γ, a characteristic impedance Z_c, and pressure and flow rate variation at a reference point x = 0.

If the mean flow is in the direction of positive lateral distance, it is an easy matter to verify that the preceding analysis is valid if x is replaced by -x in all functional entities. Thus, if mean flow is from right to left in Figure 2,

$$P(x, s) = P(0, s) \cosh \gamma x - Z_c \, Q(0, s) \sinh \gamma x , \qquad (2.32)$$

and

$$Q(x, s) = Q(0, s) \cosh \gamma x - \frac{P(0, s)}{Z_c} \sinh \gamma x . \qquad (2.33)$$

This will continue to be true in all further developments.

The foregoing analysis is based on turbulent conditions, but it holds equally well for laminar flow. For laminar flow,

$$\bar{p}_f = \frac{32 \mu \, \ell \bar{q}}{AD^2} , \qquad (2.34)$$

and n = 1. Thus,

$$R = \frac{32 \, \mu}{AD^2} \qquad (2.35)$$

where μ denotes dynamic viscosity.

For a pipe having one end closed the mean flow is zero. Here the analysis breaks down only in the determination of the frictional resistance term. It appears reasonable to assume the coefficient of resistance for this case to be the same as for laminar flow.

2.5 Frequency Response and System Parameters.

The variables $p(x, t)$ and $q(x, t)$ are considered the output of the system. The input is $p(0, t)$ and $q(0, t)$. Then, the frequency response of the system is obtained from Equations (2.30) and (2.31) by replacing s by $j\omega$. That is,

$$P(x, \omega) = P(0, \omega) \cosh \gamma x + Z_c Q(0, \omega) \sinh \gamma x, \qquad (2.36)$$

and

$$Q(x, \omega) = Q(0, \omega) \cosh \gamma x + \frac{P(0, \omega)}{Z_c} \sinh \gamma x \qquad (2.37)$$

where

$$\gamma^2 = j\omega C (R + j\omega L), \qquad (2.38)$$

and

$$Z_c^2 = \frac{R + j\omega L}{j\omega C} . \qquad (2.39)$$

The propagation coefficient, γ, can be expressed in the form $\alpha + j\beta$ by finding the roots of Equation (2.38). The attenuation constant, α, is given by

$$\alpha = \left[\frac{\omega C}{2} \left(\sqrt{R^2 + \omega^2 L^2} - \omega L \right) \right]^{\frac{1}{2}} . \qquad (2.40)$$

The phase constant, β, is given by

$$\beta = \left[\frac{\omega C}{2} \left(\sqrt{R^2 + \omega^2 L^2} + \omega L \right) \right]^{\frac{1}{2}} . \qquad (2.41)$$

The characteristic impedance, Z_c, can also be expressed in complex form by finding the roots of Equation (2.39). It is given by

$$Z_c = \frac{1}{\omega C} (\beta - j\alpha) . \qquad (2.42)$$

Expanding Equation (2.40) in powers of $\left(\frac{R}{\omega L}\right)^2$,

$$\alpha = \left\{ \frac{\omega^2 CL}{2} \left[\frac{1}{2} \left(\frac{R}{\omega L}\right)^2 - \frac{1}{8} \left(\frac{R}{\omega L}\right)^4 + \ldots \right] \right\}^{\frac{1}{2}} . \qquad (2.43)$$

If the above series is assumed sufficiently approximated by its first term, Equation (2.43) becomes

$$\alpha \sim \frac{R}{2La} . \qquad (2.44)$$

By a similar argument, Equation (2.41) becomes

$$\beta \sim \frac{\omega}{a} . \qquad (2.45)$$

Assuming α to be small as compared to β, the characteristic impedance can be considered real and given as

$$Z_c \sim La . \qquad (2.46)$$

Actual calculation of $\alpha, \beta,$ and Z_c for various systems by both exact and approximate formulation has shown the approximations to be of sufficient accuracy with the possible exception of very low frequencies [20]. Therefore, Equations (2.44), (2.45), and (2.46) will be used in further developments.

CHAPTER III

TRANSFER FUNCTION DETERMINATION

3.1 Introduction.

Using Equations (2.36) and (2.37), transfer functions relating output pressure and flow to input pressure are obtained for a single pipe and parallel pipes. Extension of the single pipe analysis to pipes in series is described. The transfer relations for single and series pipes are obtained in two different forms, both giving identical results and each having merit for different situations. The development is based on total length of a component, but can easily be generalized for any arbitrary point in the system.

3.2 Single Pipe.

Figure 3 depicts a single pipe which is the ith component of a system. The receiving and sending ends of the pipe are denoted by i and i+1, respectively. The complex pressure and flow rate variations at a point i are denoted P_i and Q_i. The absolute values of these variables are $\overline{P}_i$ and $\overline{Q}_i$.

The impedance Z_i at a point i is defined as the ratio of pressure to volume flow rate at that point, i.e.,

$$Z_i \triangleq \frac{P_i}{Q_i} \ . \tag{3.1}$$

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The absolute value of Z_i is $\overline{Z}_i$, and its phase angle is ϕ_i, i.e.,

$$Z_i = \overline{Z}_i\, e^{j\phi_i} . \tag{3.2}$$

P_{i+1}
Q_{i+1}

x_i

P_i
Q_i

flow

$\#_i$

ℓ_i

i+1 i
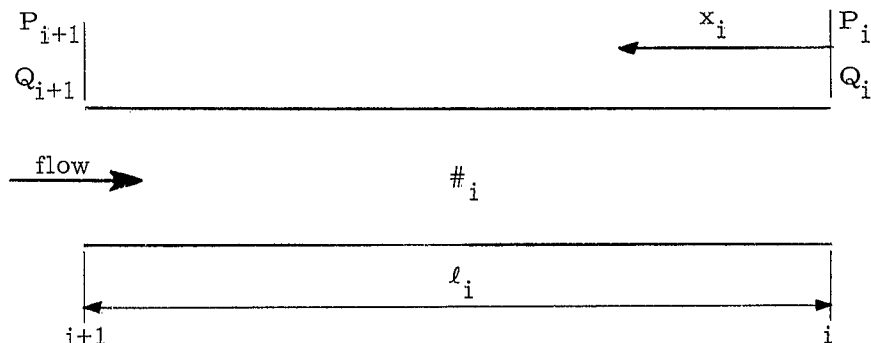
Figure 3. Single Pipe.

Denoting γ and Z_c of pipe i as γ_i and Z_{ci}, Equations (2.36) and (2.37) can be manipulated to yield

$$P_{i+1} = P_i \left[\cosh \gamma_i \ell_i + \frac{Z_{ci}}{Z_i}\, \sinh \gamma_i \ell_i \right]$$
$$= P_i G_i , \tag{3.3}$$

and

$$Q_{i+1} = P_i \left[\frac{1}{Z_i}\, \cosh \gamma_i \ell_i + \frac{1}{Z_{ci}}\, \sinh \gamma_i \ell_i \right]$$
$$= P_i H_i \tag{3.4}$$

where G_i and H_i are transfer functions. The complex functions G_i and H_i can be expressed in more suitable form for numerical use by replacing γ_i by $\alpha_i + j\beta_i$. Equations (3.3) and (3.4) represent one form of transfer function determination.

The reflection coefficient at a point i is defined as

$$\Gamma_i \triangleq \frac{Z_i - Z_{ci}}{Z_i + Z_{ci}} . \tag{3.5}$$

The magnitude of Γ_i is $\overline{\Gamma}_i$, and its phase is θ_i. Using Equation (3.5), Equations (2.36) and (2.37) are manipulated to give

$$P_{i+1} = P_i e^{\gamma_i \ell_i} \left\{ \frac{1 + \Gamma_i e^{-2\gamma_i \ell_i}}{1 + \Gamma_i} \right\}, \qquad (3.6)$$

and·

$$Q_{i+1} = P_i \frac{e^{\gamma_i \ell_i}}{Z_{ci}} \left\{ \frac{1 - \Gamma_i e^{-2\gamma_i \ell_i}}{1 + \Gamma_i} \right\}. \qquad (3.7)$$

The second useful form of transfer function determination follows from taking the absolute value of both sides of Equations (3.6) and (3.7) This gives

$$\overline{P}_{i+1} = \overline{P}_i\, e^{\alpha_i \ell_i} \left\{ \frac{1 + A_i^2 + 2A_i \cos \psi_i}{1 + \overline{\Gamma}_i^2 + 2 \overline{\Gamma}_i \cos \theta_i} \right\}^{\frac{1}{2}}$$

$$= \overline{P}_i\, \overline{G}_i, \qquad (3.8)$$

and

$$\overline{Q}_{i+1} = \overline{P}_i \frac{e^{\alpha_i \ell_i}}{Z_{ci}} \left\{ \frac{1 + A_i^2 - 2A_i \cos \psi_i}{1 + \overline{\Gamma}_i^2 + 2 \overline{\Gamma}_i \cos \theta_i} \right\}^{\frac{1}{2}}$$

$$= \overline{P}_i\, \overline{H}_i \qquad (3.9)$$

where

$$A_i = \overline{\Gamma}_i e^{-2\alpha_i \ell_i}, \qquad (3.10)$$

and

$$\psi_i = -2\beta_i \ell_i + \theta_i. \qquad (3.11)$$

The amplitude density spectrums of the pressure and flow rate at the point of $i + 1$ are obtained either by taking the absolute value of results from Equations (3.3) and (3.4) or by employing Equations (3.8) and (3.9).

3.3 Series Pipes.

It has been shown that transfer functions can be obtained relating the output at a point in a series system to the input at any other point [36, 92]. The resulting functions are quite complicated and seem to possess no advantages over the following method of analysis.

Consider the series piping system shown in Figure 4. The notation follows that of the preceding section. The logical method of analysis follows from either Equations (3.3) and (3.4) or Equations (3.8) and (3.9).
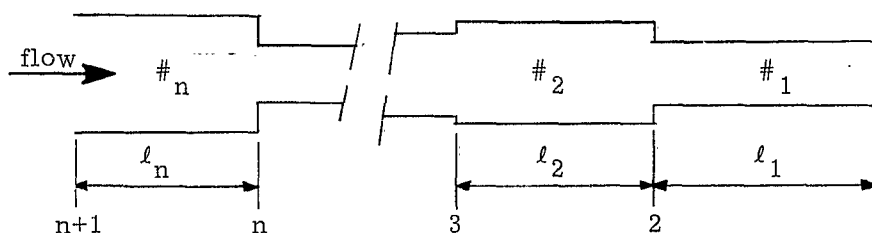
Figure 4. Series Piping System.

The analysis starts at the point 1 where end conditions are known, i.e., P_1 and Z_1 are known. Using Equations (3.3) and (3.4) or Equations (3.8) and (3.9), P_2 and Q_2 or $\overline{P}_2$ and $\overline{Q}_2$ are obtained. The procedure is repeated for each segment until the desired point is reached.

To repeat the above procedure, the impedance Z_i of each ith segment must be determined. If Equations (3.3) and (3.4) are being employed,

$$Z_i = \frac{G_{i-1}}{H_{i-1}} \quad . \tag{3.12}$$

With reference to Equations (3.8) and (3.9) ,

$$Z_i = \frac{\overline{G}_{i-1}}{\overline{H}_{i-1}} e^{j\phi_i} \tag{3.13}$$

where

$$\phi_i = \tan^{-1} \frac{2A_{i-1} \sin \psi_{i-1}}{1 - A_{i-1}^2} \tag{3.14}$$

as can be shown by manipulating Equations (3.6) and (3.7).

3.4 Parallel Pipes.

A parallel piping system is shown in Figure 5. The parallel components are not necessarily dimensionally or materially identical.
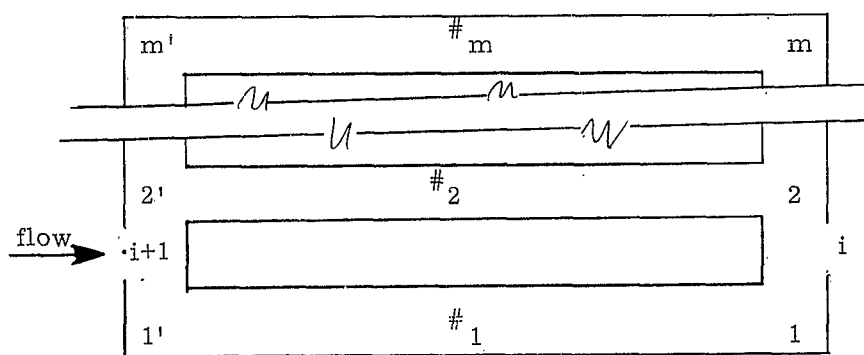
Figure 5. Parallel Piping System

The development starts with P_i and Z_i assumed known which in turn determines Q_i. The following relationships are used:

$$P_i = P_1 = P_2 = \ldots = P_m ; \tag{3.15}$$

$$Q_i = Q_1 + Q_2 + \ldots + Q_m ; \tag{3.16}$$

$$P_{i+1} = P_{1'} = P_{2'} = \ldots = P_{m'} ; \tag{3.17}$$

$$Q_{i+1} = Q_{1'} + Q_{2'} + \ldots + Q_{m'} ; \tag{3.18}$$

Using the notation

$$B_k = \cosh \gamma_k \ell_k , \tag{3.19}$$

$$T_k = \sinh \gamma_k \ell_k , \tag{3.20}$$

and

$$M_k = \frac{1}{Z_{ck}} \sinh \gamma_k \ell_k \tag{3.21}$$

where k refers to pipe number, Equations (2.36) and (2.37) give

$$P_{k'} = P_k B_k + Q_k T_k , \tag{3.22}$$

and

$$Q_{k'} = Q_k B_k + P_k M_k , \tag{3.23}$$

$k = 1, 2, \ldots, m$. Using relationships (3.15) and (3.17), subtraction of Equation (3.22) for $k = 1$ from each of the remaining Equations (3.22) yields the m-1 equations

$$T_1 Q_1 - T_k Q_k = P_i (B_k - B_1) , \tag{3.24}$$

$k = 2, 3, \ldots, m$. Adding Equations (3.23) and substituting relationships (3.15) and (3.18) gives

$$Q_{i+1} - \sum_{k=1}^{m} B_k Q_k = P_i \sum_{k=1}^{m} M_k . \tag{3.25}$$

Equations (3.16), (3.24), and (3.25) provide m+1 linearly independent equations in the m+1 unknowns Q_{i+1} and Q_k, $k=1, 2, \ldots, m$.

Thus, Q_{i+1} can be determined, and once any Q_k is found, Equation (3.22) yields P_{i+1} directly. Also, the pressure and flow rate at any point in a pipe k can be determined by finding Q_k and using Equations (2.36) and (2.37).

The results of the analysis can be expressed in the general forms

$$P_{i+1} = P_i G_i ,$$
(3.26)

and

$$Q_{i+1} = P_i H_i$$
(3.27)

where G_i and H_i are transfer functions. For m=2,

$$G_i = B_1 + \frac{T_1 \left[(B_2 - B_1) Z_i + T_2 \right]}{Z_i (T_1 + T_2)} ,$$
(3.28)

and

$$H_i = \frac{\left[-(B_2 - B_1)^2 + (M_1 + M_2)(T_1 + T_2) \right] Z_i + B_1 T_2 + B_2 T_1}{Z_i (T_1 + T_2)}$$
(3.29)

The functions G_i and H_i become increasingly complicated as m increases.

If the parallel pipes are dimensionally and materially alike,

$$\bullet \quad Q_k = \frac{1}{m} Q_i ,$$
(3.30)

and

$$Q_{k'} = \frac{1}{m} Q_{i+1} .$$
(3.31)

Then, Equations (3.22) and (3.23) give

$$P_{i+1} = P_i \left(B_k + \frac{1}{m} \frac{T_k}{Z_i} \right)$$

$$= P_i G_i ,$$
(3.32)

and

$$Q_{i+1} = P_i \left(\frac{B_k}{Z_i} + mM_k \right)$$

$$= P_i H_i$$

(3. 33)

for any $k = 1, 2, \ldots, m$.

3.5 Summary.

The results of this section make it possible to determine pressure and flow rate fluctuation at any point of a system, such as shown in Figure 1a, in terms of an input pressure and impedance at some other point of the system. The determination of this input pressure and impedance is discussed in the next chapter.

The transfer functions and usually the input involved in the analysis of a system are frequency dependent. The output of the system is then dependent on frequency; in fact, it varies considerably over relatively small frequency increments. Thus, analysis of a piping system for a large range of frequencies involves an impractical amount of numerical computation. For this reason, the possibility of finding the maximum and minimum system output is investigated in Chapter V.

CHAPTER IV

INPUT DETERMINATION

4.1 General.

Application of analysis methods developed thus far is dependent on ability to determine an input pressure and impedance at some point in a piping system. In the general sense, the input to a system is caused by a system element producing a flow disturbance. Typical flow disturbing elements are pumps and valves.

If the input is deterministic, i.e., expressible as a function of time, the Fourier transforms of functional relationships for pressure and flow rate fluctuation can usually be obtained [96]. This yields the necessary information concerning input for analysis. Two deterministic inputs often encountered in piping systems are reciprocating pumps [94, 95] and valve closures [76, 93].

In the case of centrifugal or similar type pumps, the input is non-deterministic, or random. Many other flow disturbing devices such as valves produce random inputs. Statistical methods must then be introduced in an analysis incorporating such inputs [20, 36, 92]. Statistical analysis lends itself toward uncertainty, because determination of statistical properties concerning pump and valve inputs is at best questionable.

The foregoing discussion of input determination has been directed

toward a problem of prediction, i. e. , given a system containing a flow disturbing element, what is the pressure and flow rate variation at some other point in the system? Another problem of equal importance is one of specification, i. e. , given a system, what is the allowable pressure and flow rate variation at a point in the system so as not to exceed some specified variation at another point? More specifically, given a system, what is the allowable pressure amplitude density spectrum at a point in the system so as not to exceed some specified pressure amplitude spectrum at a termination point of the system?

The methods of analysis developed in Chapter III are easily applied to the problem of specification for systems with deterministic or random inputs. The remainder of this chapter and Chapter V is directed toward problems of this type.

Assuming a pressure amplitude density spectrum is specified at a termination point of a system, the impedance at that point must be determined to employ the transfer relations developed in Chapter III. This impedance is denoted by Z_r. Closed and open end terminations are discussed in the following.

4.2 Closed End Impedance.

For a pipe with one end rigidly closed, the flow rate fluctuation at the closed end is zero. The closed end impedance is then infinite. It is also easily shown that the reflection coefficient is 1.

A stub line such as shown in Figure 1a as pipe D is a case of closed end termination. In this case, the impedance of the stub line at the junction is necessary in analysis of the complete system. From Equations (3. 3) and (3. 4), the impedance is calculated directly by

$$Z = Z_{cD} \coth \gamma_D \ell_D \quad . \tag{4.1}$$

4.3 Open End Impedance.

The impedance at an open end termination of a piping system is usually considered zero on the hypothesis that pressure fluctuation at that point is zero. This assumption is valid only for very low frequencies where the range of validity depends on pipe size.

If the velocity at the open end is considered harmonic and of constant profile, the problem becomes the classical acoustics problem of determining the impedance of a rigid circular piston set in an infinite baffle. The foregoing assumptions seem reasonable and, at least, offer a simple method of determining the open end impedance.

An excellent development of the solution to the afore mentioned problem can be found in the text by Kinsler and Frey [107]. The desired result is

$$Z_r = \frac{\rho c}{A} \left[R_1(x) + j X_1(x) \right] \tag{4.2}$$

where c is the velocity of wave propagation in the infinite medium, and

$$x = \frac{\omega D}{c} \quad . \tag{4.3}$$

The area of the open end is A, and the diameter is D. Also,

$$R_1(x) = 1 - \frac{2J_1(x)}{x} = \frac{x^2}{2 \cdot 4} - \frac{x^4}{2 \cdot 4^2 \cdot 6} + \frac{x^6}{2 \cdot 4^2 \cdot 6^2 \cdot 8} - \cdots, \tag{4.4}$$

and

$$X_1(x) = \frac{2K_1(x)}{x^2} = \frac{4}{\pi} \left[\frac{x}{3} - \frac{x^3}{3^2 \cdot 5} + \frac{x^5}{3^2 \cdot 5^2 \cdot 7} - \cdots \right] \tag{4.5}$$

where $K_1(x)$ is a related Bessel function. For small values of x,

$$R_1(x) \sim \frac{x^2}{8} \,, \qquad\qquad (4.6)$$

and

$$X_1(x) \sim \frac{4x}{3\pi} \,. \qquad\qquad (4.7)$$

The functions $R_1(x)$ and $X_1(x)$ are shown graphically and tabulated numerically in Kinsler and Frey's book. The graphical representation is reproduced in Chapter VI, Figure 8. The functions are also approximated for computational purposes in that chapter.

Although the open end impedance depends on frequency, it can be considered essentially constant over frequency bands. This fact is used in the next chapter.

CHAPTER V

MAXIMUM AND MINIMUM
ALLOWABLE PRESSURE SPECTRUM

5.1 Introduction.

An allowable pressure level at some point in a piping system for a given frequency can now be determined. To obtain a complete allowable pressure spectrum for a sizable frequency range, a large number of calculations must be made. For this reason, the problem of determining a maximum and minimum allowable pressure spectrum is now approached.

5.2 General.

Systems to be studied will have open end termination as discussed in Section 4.3 of this report. This type of termination is normally the case encountered in sea connected systems. The specified pressure amplitude spectrum at the termination is assumed constant over a frequency band. This will allow easy adjustment to the true overboard specifications of pressure. Thus, the specified pressure spectrum and impedance at the termination are considered constant over a frequency band where the bands cover the frequency range of interest, e.g., one-third octave bands from 0 to 10,000 cps.

The object of the investigation is then to determine the maximum and minimum allowable pressure levels at some point in the system

for each frequency band. Once these levels are determined, the following remarks can be made about the actual pressure amplitude occurring at the point for a given frequency in a band.

If the pressure amplitude is greater than the maximum level, the pressure amplitude at the termination is greater than the specified amplitude. If the pressure is less than the minimum level, the termination amplitude is less than the specified pressure. If the pressure is between the maximum and minimum levels, the termination pressure can be either above or below the specified pressure amplitude. For this case, the allowable pressure for the frequency in question must be calculated to reach a conclusion.

If these maximum and minimum levels can be determined, the problem of specification is reduced considerably. Much effort by the authors has been devoted to determining these extremes. It has been concluded that the maximum and minimum levels cannot be determined analytically for any but the simplest system, i.e., a single pipe.

The next section describes a method for analytically determining the desired extremes for a single pipe. Section 5.4 discusses the problems encountered in attempts to extend the analysis to pipes in series. The developments are based on simple arguments. Methods of variational calculus attempted provided no information other than the obvious.

5.3 Single Pipe.

The single pipe system is shown in Figure 6. Equations (3.8) and (3.9) give

$$\bar{P}_2 = \bar{P}_r e^{\alpha_1 \ell_1} \left\{ \frac{1 + A_1^2 + 2A_1 \cos \psi_1}{1 + \bar{\Gamma}_r^2 + 2\bar{\Gamma}_r \cos \theta_r} \right\}^{\frac{1}{2}} , \qquad (5.1)$$

and

$$\overline{Q}_2 = \overline{P}_r \frac{e^{\alpha_1 \ell_1}}{Z_{c1}} \left\{ \frac{1 + A_1^2 - 2A_1 \cos \psi_1}{1 + \overline{\Gamma}_r^2 + 2\overline{\Gamma}_r \cos \theta_r} \right\}^{\frac{1}{2}} . \tag{5.2}$$
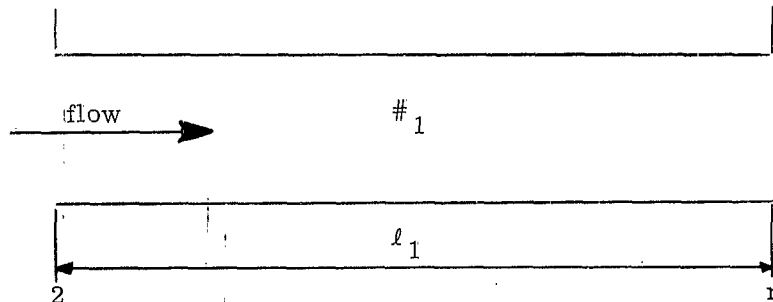

Figure 6. Single Pipe.

Since all end conditions are assumed constant over a frequency band, the only frequency dependent parameter in Equations (5.1) and (5.2) is β_1, or $\psi_1 = -2\beta_1 \ell_1 + \theta_r$. Obviously, the maximum $\overline{P}_2$ occurs when $\cos \psi_1 = 1$, and the minimum $\overline{P}_2$ occurs when $\cos \psi_1 = -1$. Similarly, $\cos \psi_1 = 1$ gives minimum $\overline{Q}_2$, and $\cos \psi_1 = -1$ gives maximum $\overline{Q}_2$.

The desired maximum and minimum pressure levels for a frequency band are then

$$\overline{P}_{2max} = \frac{\overline{P}_r e^{\alpha_1 \ell_1} (1+A_1)}{\left[1 + \overline{\Gamma}_r^2 + 2\overline{\Gamma}_r \cos \theta_r \right]^{\frac{1}{2}}} , \tag{5.3}$$

and

$$\overline{P}_{2min} = \frac{\overline{P}_r e^{\alpha_1 \ell_1} (1-A_1)}{\left[1 + \overline{\Gamma}_r^2 + 2\overline{\Gamma}_r \cos \theta_r \right]^{\frac{1}{2}}} . \tag{5.4}$$

The number of calculations required to cover the frequency range of interest depends only on the number of bands chosen to cover the range.

For a narrow bandwidth, there is a possibility the values of $\cos \psi_1$ used to obtain Equations (5.3) and (5.4) do not occur in the band. If this discrepancy cannot be allowed, it is an easy matter to determine the maximum and minimum values of $\cos \psi_1$ for the band and use them in Equations (5.1) and (5.2).

5.4 Series Pipes.

Two pipes in series are shown in Figure 7. The maximum and minimum allowable pressure levels for a particular frequency band at point 2 are computed from Equations (5.3) and (5.4). The problem is to determine the extreme pressure levels for the same frequency band at point 3.
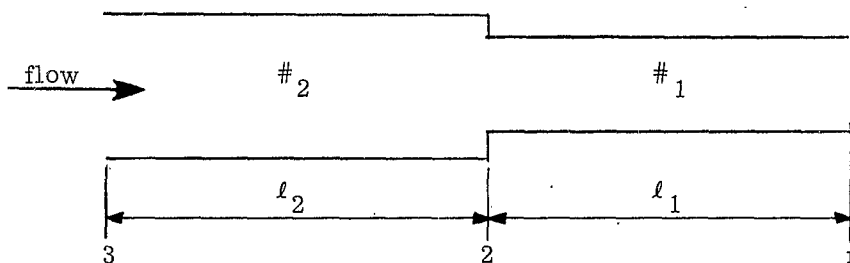
Figure 7. Two Pipes in Series.

Equation (3.8) gives

$$\overline{P}_3 = \overline{P}_2 \, e^{\alpha_2 \ell_2} \left\{ \frac{1 + A_2^2 + 2A_2 \cos \psi_2}{1 + \overline{\Gamma}_2^2 + 2\overline{\Gamma}_2 \cos \theta_2} \right\}^{\frac{1}{2}} . \qquad (5.5)$$

Unlike the termination impedance, Z_2 cannot be considered constant over the frequency band. In fact, its rate of change with respect to frequency is great. Thus, for all but very narrow bandwidths, $\overline{\Gamma}_2$

ranges from zero to 1, and θ_2 will range from zero to π in the frequency band.

The maximum $\overline{P}_3$ obviously occurs under the following conditions:

1. $\overline{P}_2$ is a maximum.

2. $\cos \psi_2 = 1$.

3. $\cos \theta_2 = -1$.

4. $\overline{\Gamma}_2$ is a maximum.

Similarly, minimum $\overline{P}_3$ will occur under the conditions:

1. $\overline{P}_2$ is a minimum.

2. $\cos \psi_2 = -1$.

3. $\cos \theta_2 = 1$.

4. $\overline{\Gamma}_2$ is a maximum.

It can be shown in both cases that conditions 1, 2, and 4 cannot occur at the same frequency. Furthermore, the possibility of condition 2 occurring simultaneously with even one of the other three conditions is very remote.

It was hoped that even with the stated discrepancies the conditions might give levels which would provide an estimation of the actual extremes. Example 2 of Chapter VI discounts this idea by showing the levels to be completely unreasonable. Successive application of the conditions to series systems with more than two pipes magnifies the factor of unreasonability.

In conclusion, it seems the analytical unpredictability of the junction impedance functions $\overline{\Gamma}$ and θ relative to the function ψ defies analytical determination of maximum and minimum allowable pressure levels for complex systems.

CHAPTER VI

EXAMPLES

6.1 Introduction.

Application of piping system analysis methods developed in Chapters I-V is illustrated by three numerical examples. The examples are oriented toward the problem of specification. The first example deals with open end impedance calculation. Determination of maximum and minimum allowable pressure levels for a series piping system is attempted in the second example. The third example illustrates the method of complete analysis for a relatively simple, but practical, piping system.

The pound-foot-second system of units is used throughout. All pressure amplitudes are expressed in decibels by the conversion

$$db = 20 \log_{10} \frac{P}{P_o} \tag{6.1}$$

where P denotes pressure pulsation magnitude in psf and the reference pressure $P_o = 4.18 \times 10^{-7}$ psf (.0002 dynes per cm^2.). The frequency range of interest is 0 to 10,000 cps in each example.

The specified termination pressure amplitude in the second and third examples is taken as zero decibels over the complete frequency range. Results of the examples can then be added to any other specified level in decibels to obtain allowable levels for the desired specification.

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The liquid considered has the following physical properties:

$$\rho = 2.0 \text{ slugs/ft}^3 .;$$

$$\mu = 2.4 \times 10^{-5} \text{ lb-sec/ft}^2 .;$$

$$c = 4900 \text{ ft/sec.};$$

$$K = 4.75 \times 10^7 \text{ psf.}$$

The elastic modulus of the pipe material is 2×10^9 psf. The constant n is taken to be 1.8 for all pipes.

6.2 Example 1. Open End Impedance.

The purpose of this example is to numerically show the frequency characteristics of the open end impedance discussed in Section 4.3. The impedance functions $R_1(x)$ and $X_1(x)$ defined by Equations (4.4) and (4.5) are shown graphically in Figure 8 as reproduced from Kinsler and Frey [107].

For digital computation purposes, approximating polynomials for $0 \leq x \leq 8$ were calculated from numerical data in Kinsler and Frey. These polynomials are

$$\begin{aligned}
R_1(x) &= (0.124324) x^2 - (5.051 \times 10^{-3}) x^4 + \\
&\quad (9.618 \times 10^{-5}) x^6 - (9.199 \times 10^{-7}) x^8 + \\
&\quad (3.621 \times 10^{-9}) x^{10} ,
\end{aligned} \qquad (6.2)$$

and

$$\begin{aligned}
X_1(x) &= (0.4220302) x - (2.73694 \times 10^{-2}) x^3 + \\
&\quad (7.08794 \times 10^{-4}) x^5 - (8.45110 \times 10^{-6}) x^7 + \\
&\quad (3.91750 \times 10^{-8}) x^9 .
\end{aligned}$$

Each polynomial has a standard error of estimate of 0.0016 for 28 points.

Equations (6.2) and (6.3) were used in Equation (4.2) to calculate the open end impedance of a 5.157" inside diameter pipe. The reflection
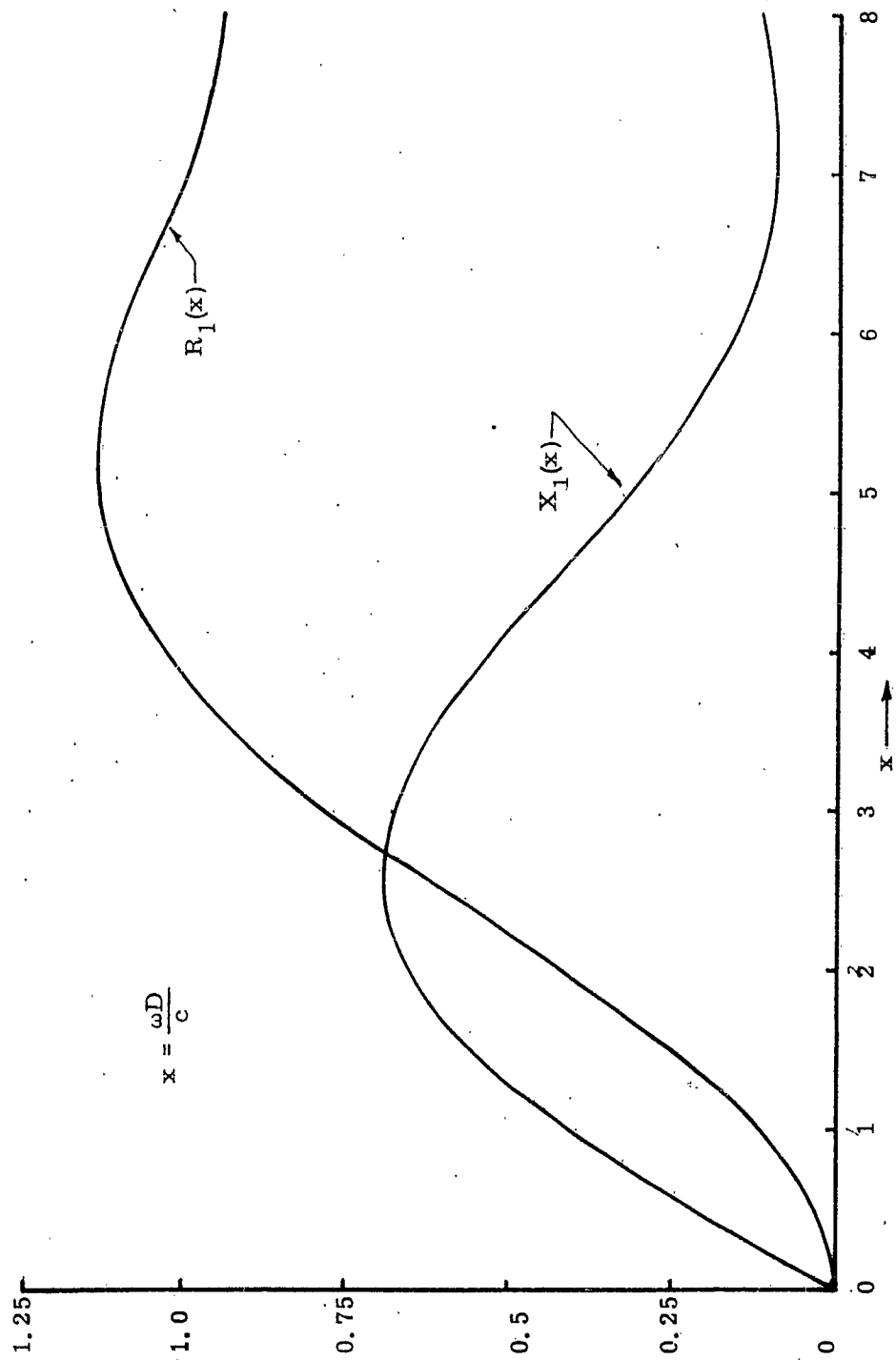
$$x = \frac{\omega D}{c}$$
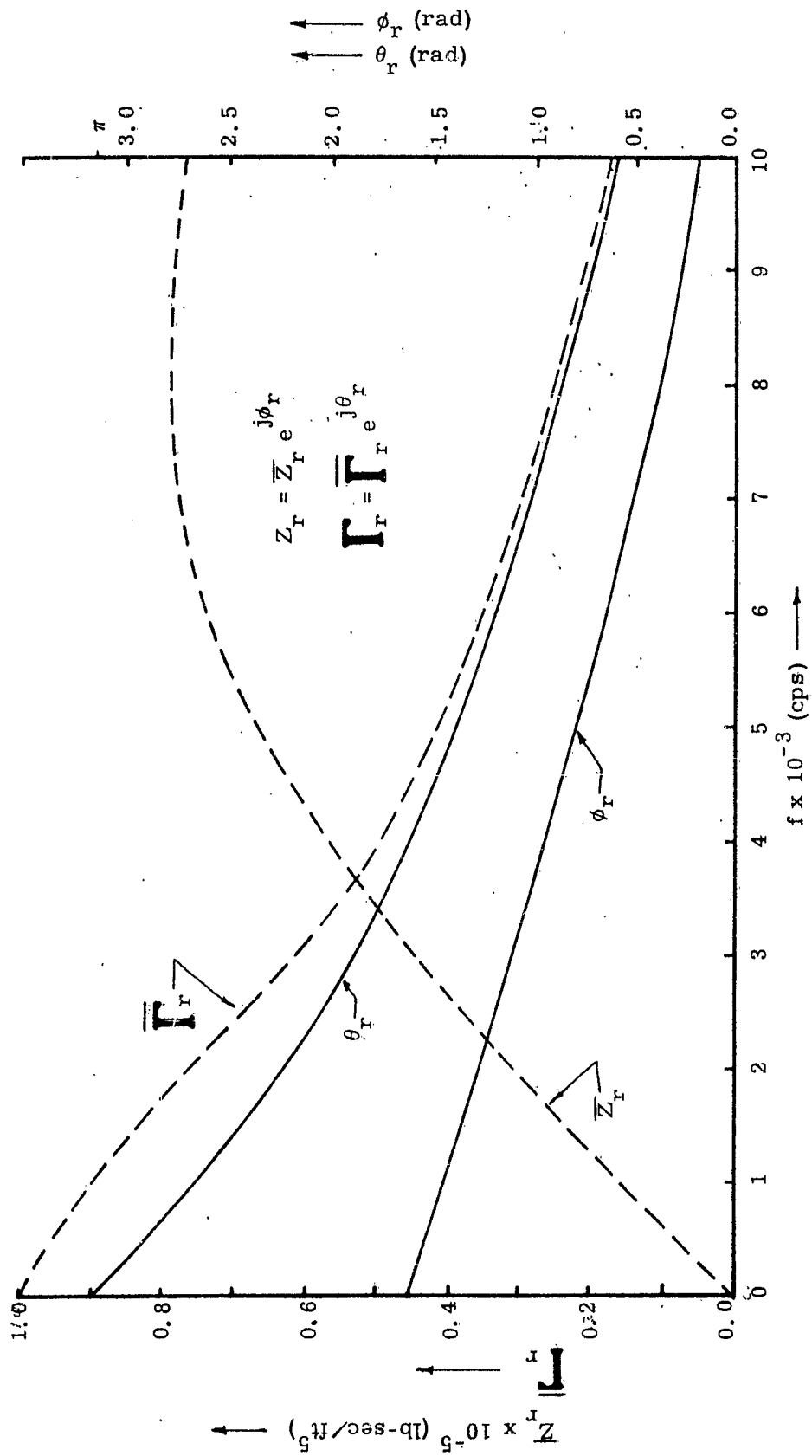
Figure 8. Open End Impedance Functions, Example 1.

Figure 9. Open End Impedance and Reflection Coefficient for 5.157" Inside Diameter Pipe, a = 4200 ft/sec., Example 1.

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coefficient of the open end was computed from Equation (3.5) where the velocity of propagation in the pipe was taken to be 4200 ft/sec. Results are shown in Figure 9.

6.3 Example 2. Maximum and Minimum Analysis.

The series piping system shown in Figure 10 is used to illustrate the methods of finding allowable maximum and minimum pressure levels discussed in Chapter V. Pertinent system data is provided by the figure. Pipe diameters are assumed such that the velocities of propagation are as given.

The input impedance was calculated in the previous section. For this example, the reflection coefficient was assumed constant over one-third octave filter bands as shown in Figure 11. Using these constant values, $\bar{P}_{2max}$ and $\bar{P}_{2min}$ were computed from Equations (5.3) and (5.4) where $\bar{P}_r$ is taken as zero decibels. Pressure levels at the points 3 and 4 were calculated on the basis of conditions outlined in section (5.4).

Results of the analysis are given in Figures 12a and 12b. As expected, the allowable levels at points 3 and 4 are unreasonable.

6.4 Example 3. Complete Analysis.

Figure 13 depicts a suction line of a piping system. System data is given on the figure. The analysis objective is to determine the allowable pressure amplitude spectrum at the pump so that the pressure amplitude spectrum of zero decibels at the terminus is not exceeded.

The strainer plate at the termination poses a somewhat different situation than previously discussed. It is evident that adding the hole impedances as parallel impedances will give the total impedance Z_r.

open end

#1

$\ell_1 = 20$ ft.

$D_1 = 5.157$ in.

$a_1 = 4200$ ft/sec.

$Z_{c1} = 57,927$ lb-sec/ft^5.

$f_1 = 0.018$

$\alpha_1 = 4.14 \times 10^{-5}$/ft.

#2

$\ell_2 = 20$ ft.

$D_2 = 6.025$ in.

$a_2 = 4100$ ft/sec.

$Z_{c2} = 41,435$ lb-sec/ft^5.

$f_2 = 0.019$

$\alpha_2 = 2.80 \times 10^{-5}$/ft.

#3

$\ell_3 = 20$ ft.

$D_3 = 3.67$ in.

$a_3 = 4300$ ft/sec.

$Z_{c3} = 117,166$ lb-sec/ft^5.

$f_3 = 0.0178$

$\alpha_3 = 11.08 \times 10^{-5}$/ft.

flow

$\overline{q} = 600$ gpm

r

2

3

4
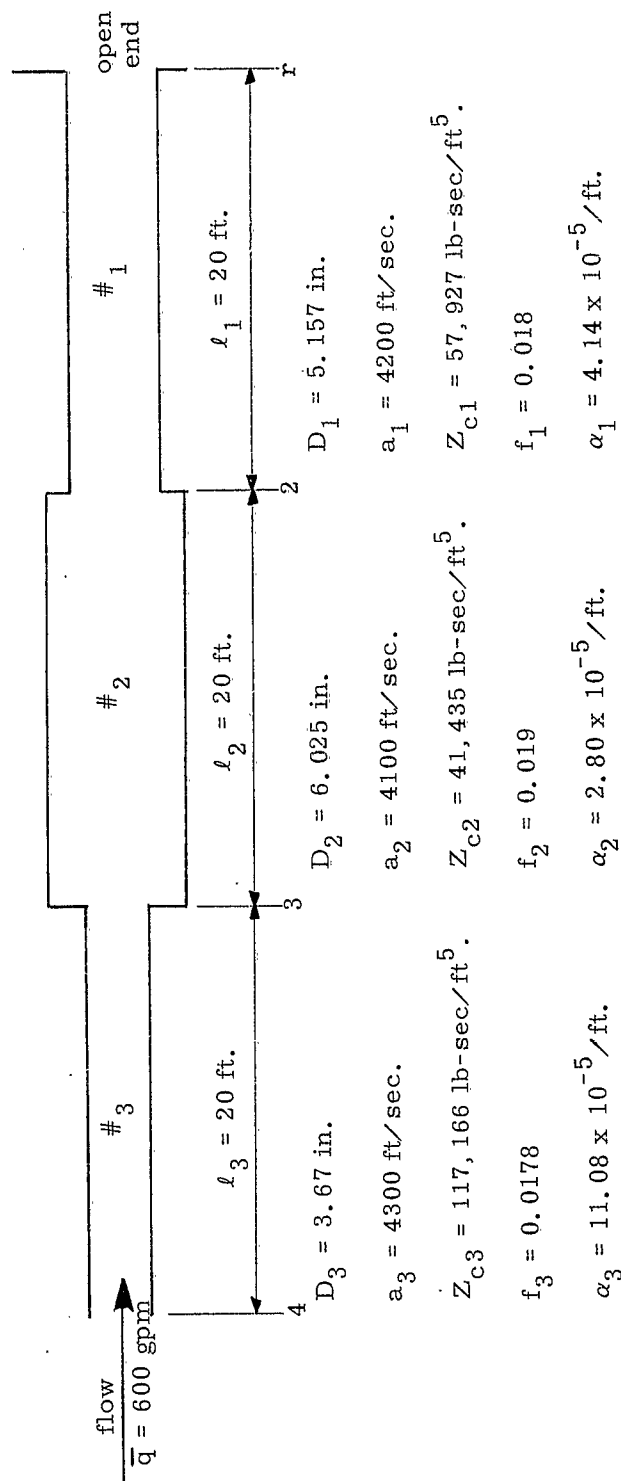
Figure 10. Series Piping System to Illustrate Maximum and Minimum Analysis, Example 2.
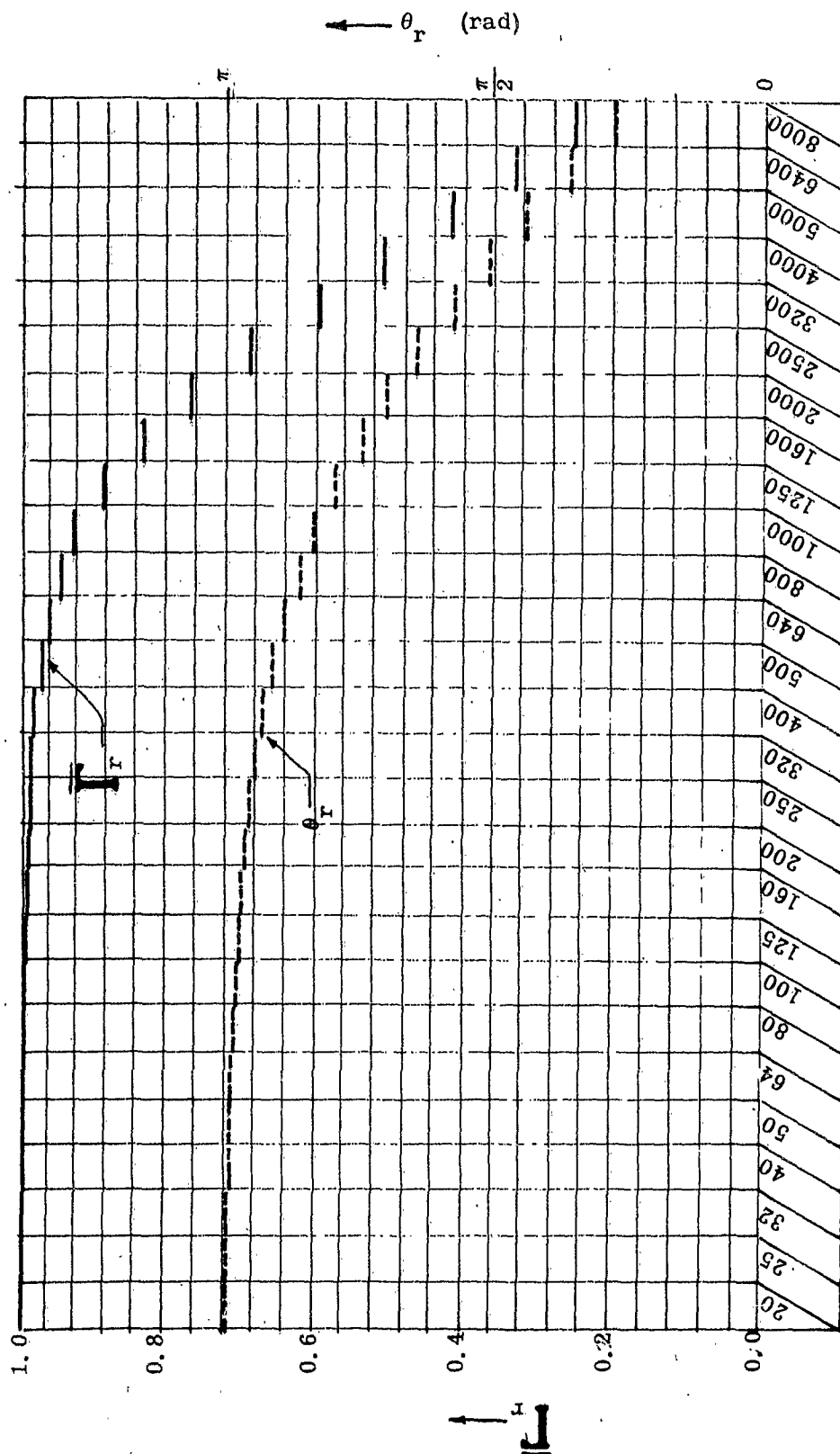
Figure 11. Approximate Open End Reflection Coefficient for 5.157" Inside Diameter Pipe, a = 4200 ft/sec., Example 2.
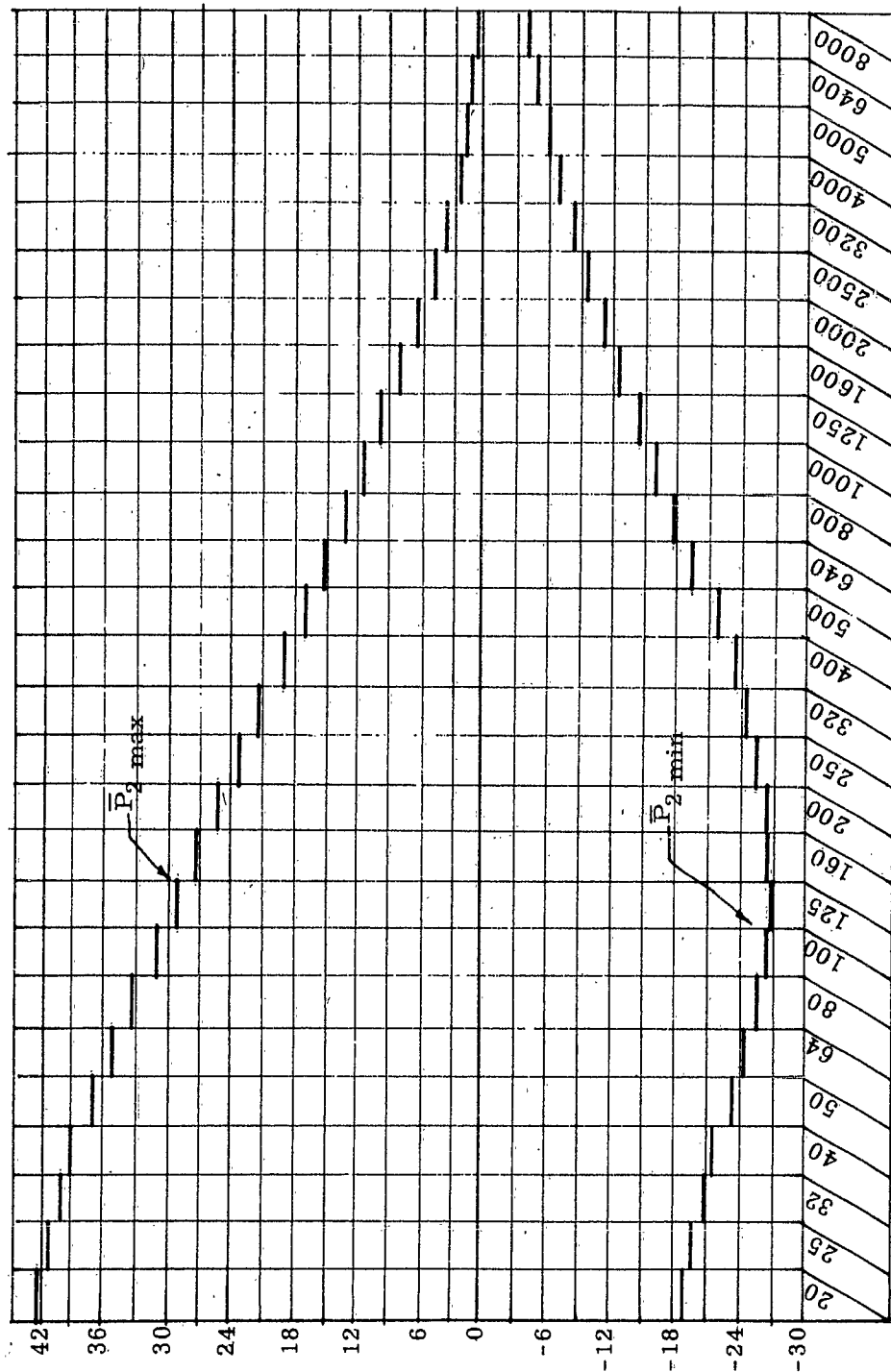
Figure 12a. Maximum and Minimum Allowable Pressure Levels at Point 2, Example 2.

47
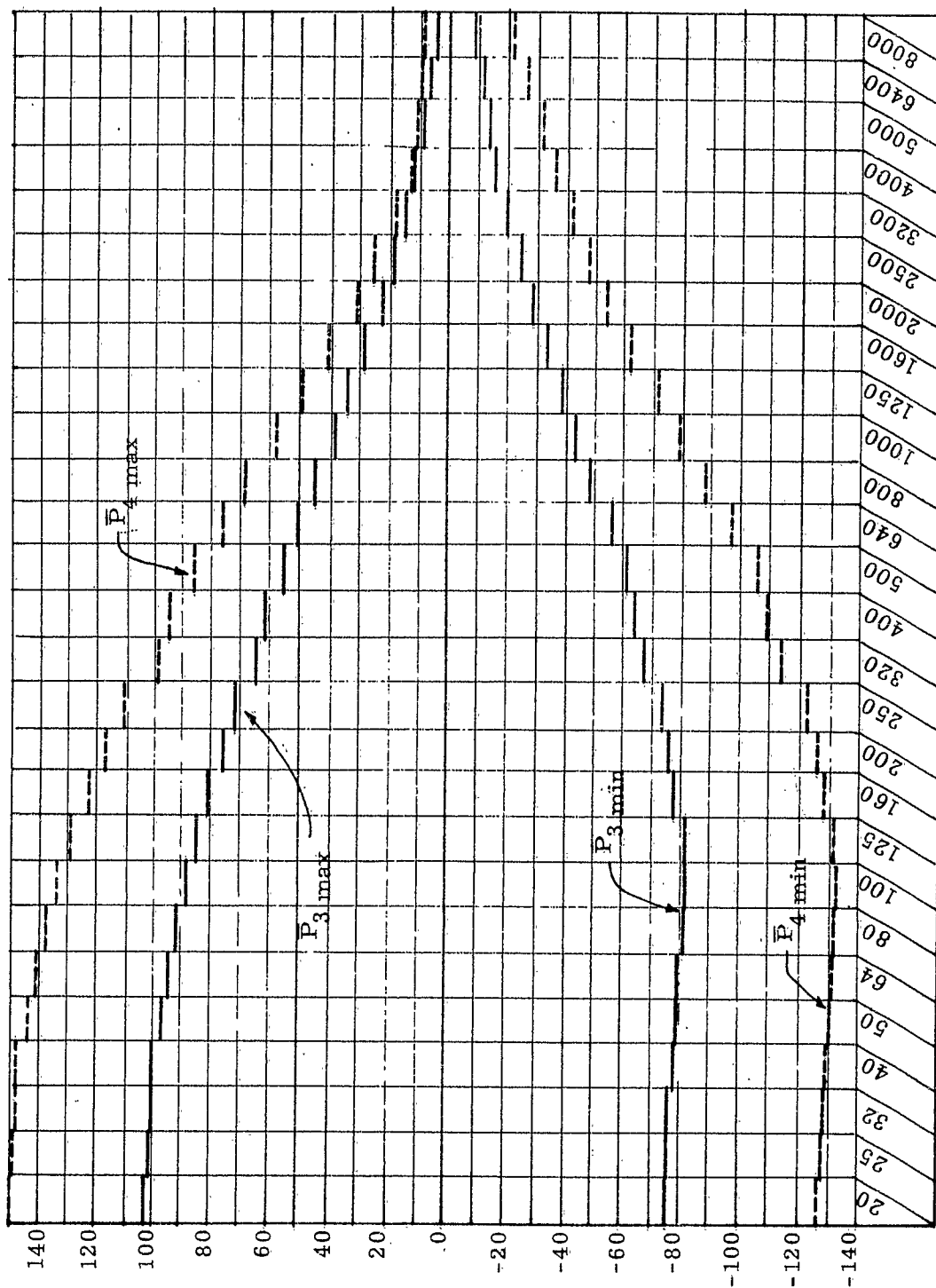
Figure 12b.   Maximum and Minimum Allowable Pressure Levels at Points 3 and 4, Example 2.

48
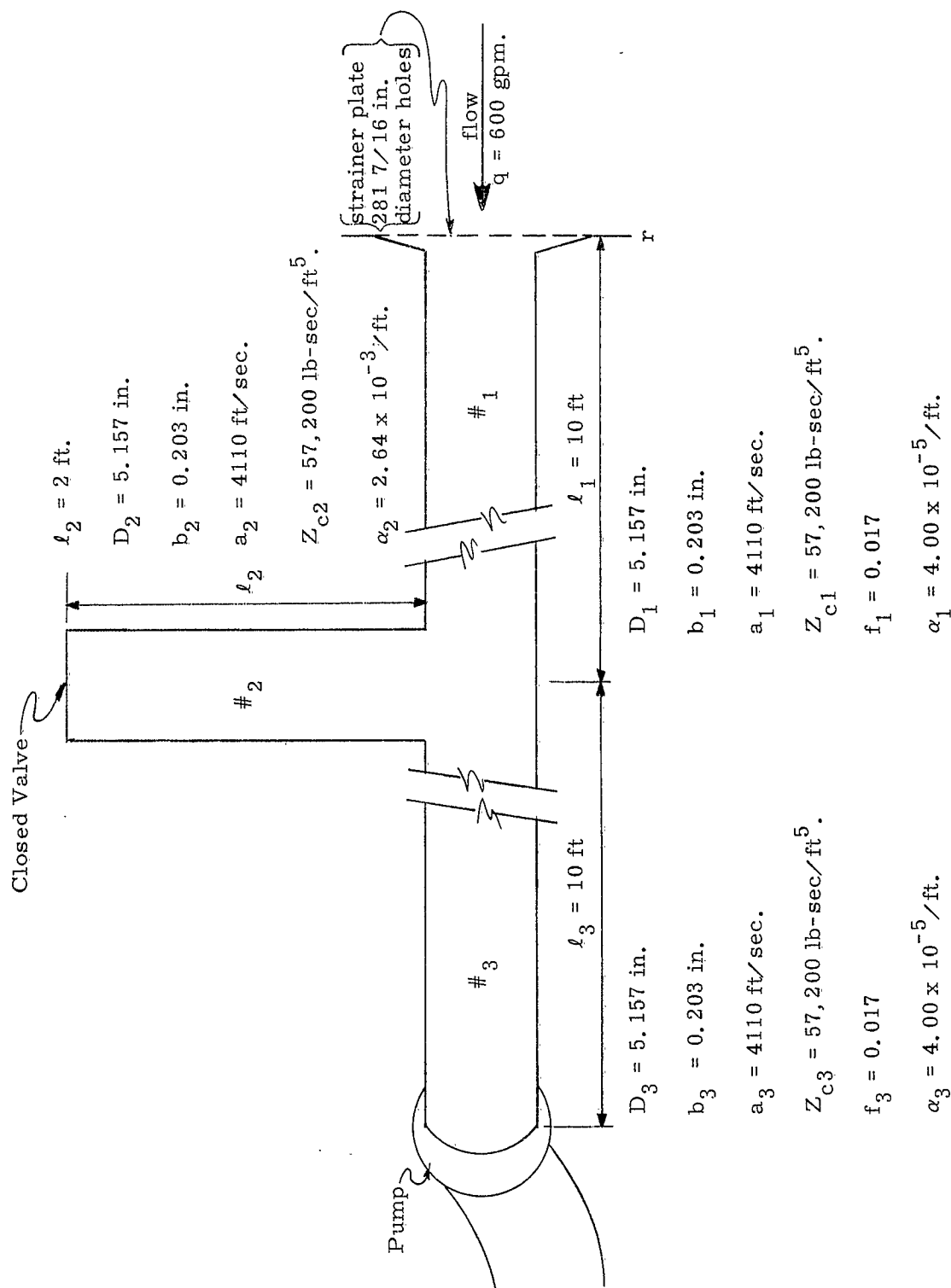
Figure 13. Suction Line to Illustrate Complete Analysis Procedure, Example 3.

$\ell_2 = 2$ ft.

$D_2 = 5.157$ in.

$b_2 = 0.203$ in.

$a_2 = 4110$ ft/sec.

$Z_{c2} = 57,200$ lb-sec/ft^5.

$\alpha_2 = 2.64 \times 10^{-3}$/ft.

$\ell_1 = 10$ ft

$D_1 = 5.157$ in.

$b_1 = 0.203$ in.

$a_1 = 4110$ ft/sec.

$Z_{c1} = 57,200$ lb-sec/ft^5

$f_1 = 0.017$

$\alpha_1 = 4.00 \times 10^{-5}$/ft.

$\ell_3 = 10$ ft

$D_3 = 5.157$ in.

$b_3 = 0.203$ in.

$a_3 = 4110$ ft/sec.

$Z_{c3} = 57,200$ lb-sec/ft^5.

$f_3 = 0.017$

$\alpha_3 = 4.00 \times 10^{-5}$/ft.

strainer plate
281 7/16 in.
diameter holes

flow
q = 600 gpm.

Closed Valve

Pump

If the hole impedances are identical, Z_r is found by dividing the impedance of one hole by the total number of holes. The problem is then to determine the impedance of each hole. In this sample, each hole of the strainer plate is assumed to have the same impedance which is given by the method of open end impedance determination discussed in Section 4.3. The physical situation violates the infinite baffle condition imposed on this method, but this discrepancy does not seem to merit a more involved study at this time. The transition in area from the strainer plate to the first pipe is neglected.

Actual computations needed for analysis are described by the FORTRAN program shown in Figure 14. Necessary explanations accompany the program. Relationships coming either in full or in part from equations in preceding chapters are noted. Necessary revisions of these equations due to mean flow in the direction of positive lateral direction have been made as outlined in Section 2.4. Erroneously, the velocity of propagation, c, was taken as 4100 ft/sec in the calculations.

For qualitative results, computations for frequency increments not to exceed 4 cps were found to suffice. For frequency ranges of special interest, the increment of computation was reduced to 0.5 cps.

The allowable pressure spectrum at the pump was found to be somewhat periodic. A typical segment of the spectrum is shown in Figure 15. The critical part of this segment is in the frequency range 4500 $<f<$ 4750 cps. It was found that the spectrum contained waveforms almost identical to the critical part of this segment every 1050-1100 cps; the first such waveform occurring at approximately 500 cps.

It was also found that all "local" absolute maximums and minimums occurred in these critical parts of the spectrum. Thus, to

obtain the maximum and minimum allowable pressure levels sought

in Chapter V, calculations are needed only for these critical ranges of

frequency. Figure 16 shows the maximums and minimums obtained

in this manner.

```
     DIMENSION ZC(3),V(3),EL(3),ALF(3),SINH(3),COSH(3)
40   PI=3.1415926
     READ 1,CON1,CON2
     DO 7 I=1,3
     READ 1,ZC(I),V(I),EL(I),ALF(I)
     ARG=ALF(I)*EL(I)
     E=EXP(ARG)
     SINH(I)=(E-1.0/E)/2.0
7    COSH(I)=(E+1.0/E)/2.0
     IF (SENSE SWITCH 2) 4,3
3    READ 1,C,DC,CM
     GO TO 5
4    ACCEPT 1,C
5    W=C*2.0*PI
     X2=CON1*C**2
     Y2=CON2*C
     I=1
     EXECUTE PROCEDURE 10
     X1=SH*ZC(I)
     Y1=SHJ*ZC(I)
     EXECUTE PROCEDURE 2
     EXECUTE PROCEDURE 30
     G1DB=20.0*0.43429448*LOG(GAB)
     X1=CH
     Y1=CHJ
     EXECUTE PROCEDURE 2
     X2=X3-SH/ZC(I)
     Y2=Y3-SHJ/ZC(I)
     X1=G
     Y1=GJ
     EXECUTE PROCEDURE 2
     Z1=X3
     Z1J=Y3
     I=2
     EXECUTE PROCEDURE 10
     X1=ZC(I)*CH
     Y1=ZC(I)*CHJ
     X2=SH
     Y2=SHJ
     EXECUTE PROCEDURE 2
     Z2=-X3
     Z2J=-Y3
     X1=Z1*Z2-Z1J*Z2J
     Y1=Z1*Z2J+Z2*Z1J
     X2=Z1+Z2
     Y2=Z1J+Z2J
     EXECUTE PROCEDURE 2
```

Right column annotations:

$PI = \pi$

$CON1 = \dfrac{\rho c}{A}\left[\dfrac{1}{8}\left(\dfrac{D}{C}\right)^2\right]\dfrac{1}{281}$ $CON2 = \dfrac{\rho c}{A}\left[\dfrac{4}{3\pi}\left(\dfrac{D}{C}\right)\right]\dfrac{1}{281}$ Eq. (4.2)

$ZC(I) = Z_{ci}$, $V(I) = a_i$, $EL(I) = \ell_i$, $ALF(I) = \gamma_i$; $i = 1,2,3$

$ARG = \alpha_i \ell_i$

$SINH(I) = \sinh \alpha_i \ell_i$

$COSH(I) = \cosh \alpha_i \ell_i$

$C = f(cps)$, $DC = \Delta f$, $CM = f_{max}$.

Accept arbitrary f(cps) from console typewriter

$W = \omega = 2\pi f$

$X2 = \text{Re}\left\{\text{Impedance at the sea}\right\} = 1.09 \times 10^{-5} f^2$ Eq. (4.2)

$Y2 = \text{Im}\left\{\text{Impedance at the sea}\right\} = 0.663 f$

Pipe Number 1

$X1 = (Z_{c1})\,\text{Re}\left\{\sinh \gamma_1 \ell_1\right\}$ Eq. (3.3)

$Y1 = (Z_{c1})\,\text{Im}\left\{\sinh \gamma_1 \ell_1\right\}$

$G1DB = $ gain in db across pipe no. 1   Eq. (6.1)

$X1 = \text{Re}\left\{\cosh \gamma_1 \ell_1\right\}$ Eq. (4.2)

$Y1 = \text{Im}\left\{\cosh \gamma_1 \ell_1\right\}$

$Z1 = \text{Re}\left\{\text{Impedance at junction, pipe no. 1}\right\}$ Eq. (3.1)

$Z1J = \text{Im}\left\{\text{Impedance at junction, pipe no. 1}\right\}$

Pipe Number 2

$Z2 = \text{Re}\left\{\text{Impedance at junction, pipe no. 2}\right\}$ Eq. (4.1)

$Z2J = \text{Im}\left\{\text{Impedance at junction, pipe no. 2}\right\}$

$\dfrac{(X1) + j(Y1)}{(X2) + j(Y2)} = $ Impedance at junction, pipe no. 3   Eq. (3.1)

Figure 14.   FORTRAN Program for Complete Analysis Procedure, Example 3.

```
I=3
EXECUTE PROCEDURE 10
X1=SH*ZC(I)
Y1=SHJ*ZC(I)
X2=X3
Y2=Y3
EXECUTE PROCEDURE 2
EXECUTE PROCEDURE 30
G2DB=20.0*0.43429448*LOG(GAB)
G=G1DB+G2DB
IF (SENSE SWITCH 1) 20,21
20 PUNCH 13,G1DB,G2DB,G,C
GO TO 22
21 TYPE 13,G1DB,G2DB,G,C
22 IF (SENSE SWITCH 2) 4,23
23 C=C+DC
IF (C-CM) 5,5,40
BEGIN PROCEDURE 2
Q=X2*X2+Y2*Y2
X3=(X1*X2+Y1*Y2)/Q
Y3=(X2*Y1-X1*Y2)/Q
END PROCEDURE 2
BEGIN PROCEDURE 30
G=CH-X3
GJ=CHJ-Y3
ARG=G*G+GJ*GJ
GAB=SQR(ARG)
END PROCEDURE 30
BEGIN PROCEDURE 10
B=W*EL(I)/V(I)
SB=SIN(B)
CB=COS(B)
CH=COSH(I)*CB
CHJ=SINH(I)*SB
SH=SINH(I)*CB
SHJ=COSH(I)*SB
END PROCEDURE 10
1 FORMAT (E10.3,E10.3,E10.3,E10.3)
13 FORMAT (F7.1,F7.1,F7.1,3X F8.1)
END
```

Pipe Number 3

$X1 = (Z_{c3})$ Re $\left\{\sinh \gamma_3 \ell_3\right\}$ Eq. (3.3)

$Y1 = (Z_{c3})$ Im $\left\{\sinh \gamma_3 \ell_3\right\}$

$X2 =$ Re. $\left\{$Impedance at junction, pipe no. 3$\right\}$ Eq. (3.1)

$Y2 =$ Im $\left\{$Impedance at junction, pipe no. 3$\right\}$

G2DB = gain in db across pipe no. 3   Eq. (6.1)

G = system gain in db

Increment frequency

Test for maximum frequency

$$(X3) + j(Y3) = \frac{(X1) + j(Y1)}{(X2) + j(Y2)}$$

GAB = $\left|$Pressure gain$\right|$

$B = \beta_i \ell_i$

$SB = \sin (\beta_i \ell_i)$

$CB = \cos (\beta_i \ell_i)$

$CH =$ Re $\left\{\cosh \gamma_i \ell_i\right\}$ Eq. (3.3)

$CHJ =$ Im $\left\{\cosh \gamma_i \ell_i\right\}$

$SH =$ Re $\left\{\sinh \gamma_i \ell_i\right\}$

$SHJ =$ Im $\left\{\sinh \gamma_i \ell_i\right\}$

Input, output format control

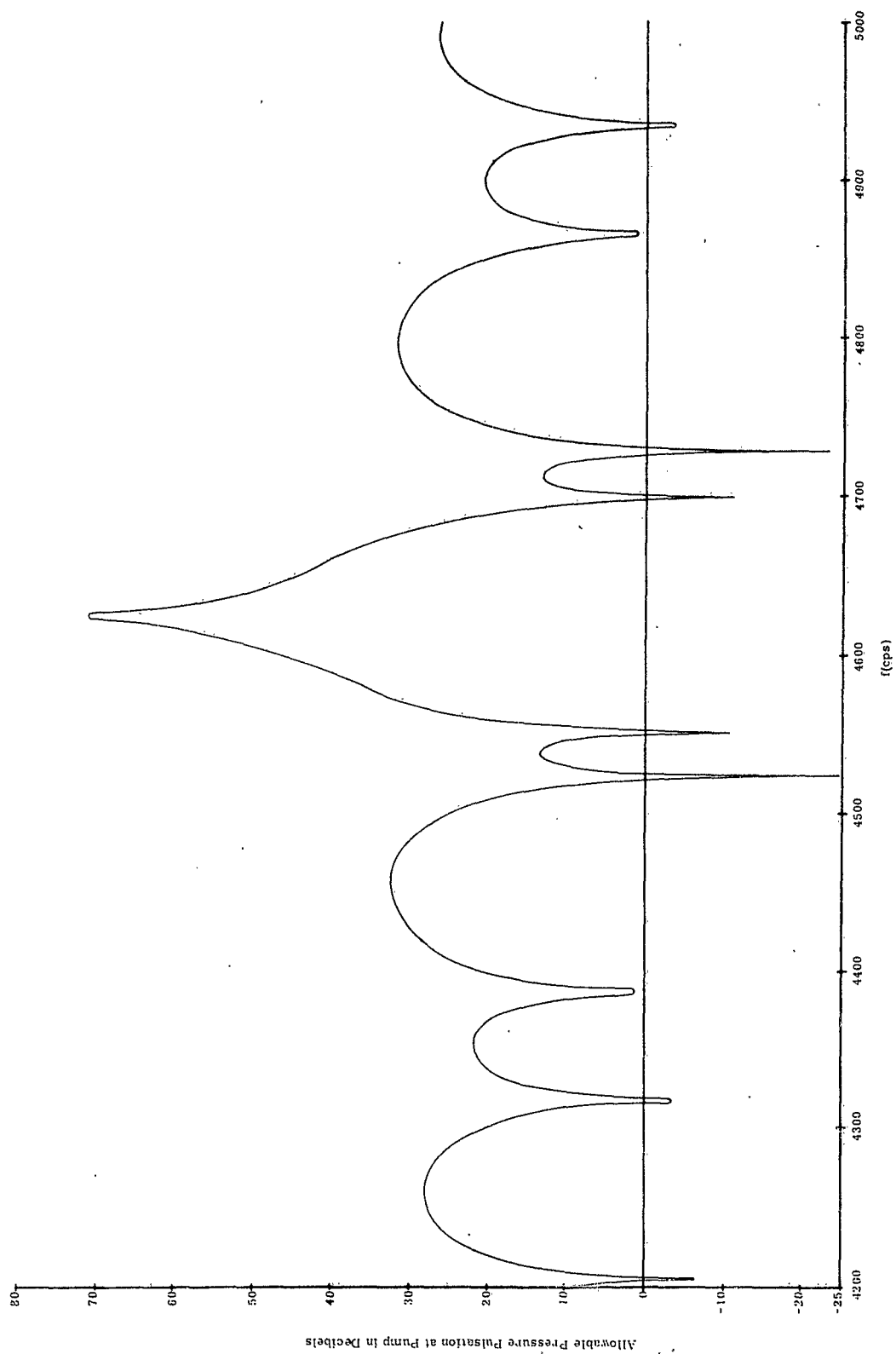Figure 14. FORTRAN Program for Complete Analysis Procedure, Example 3.

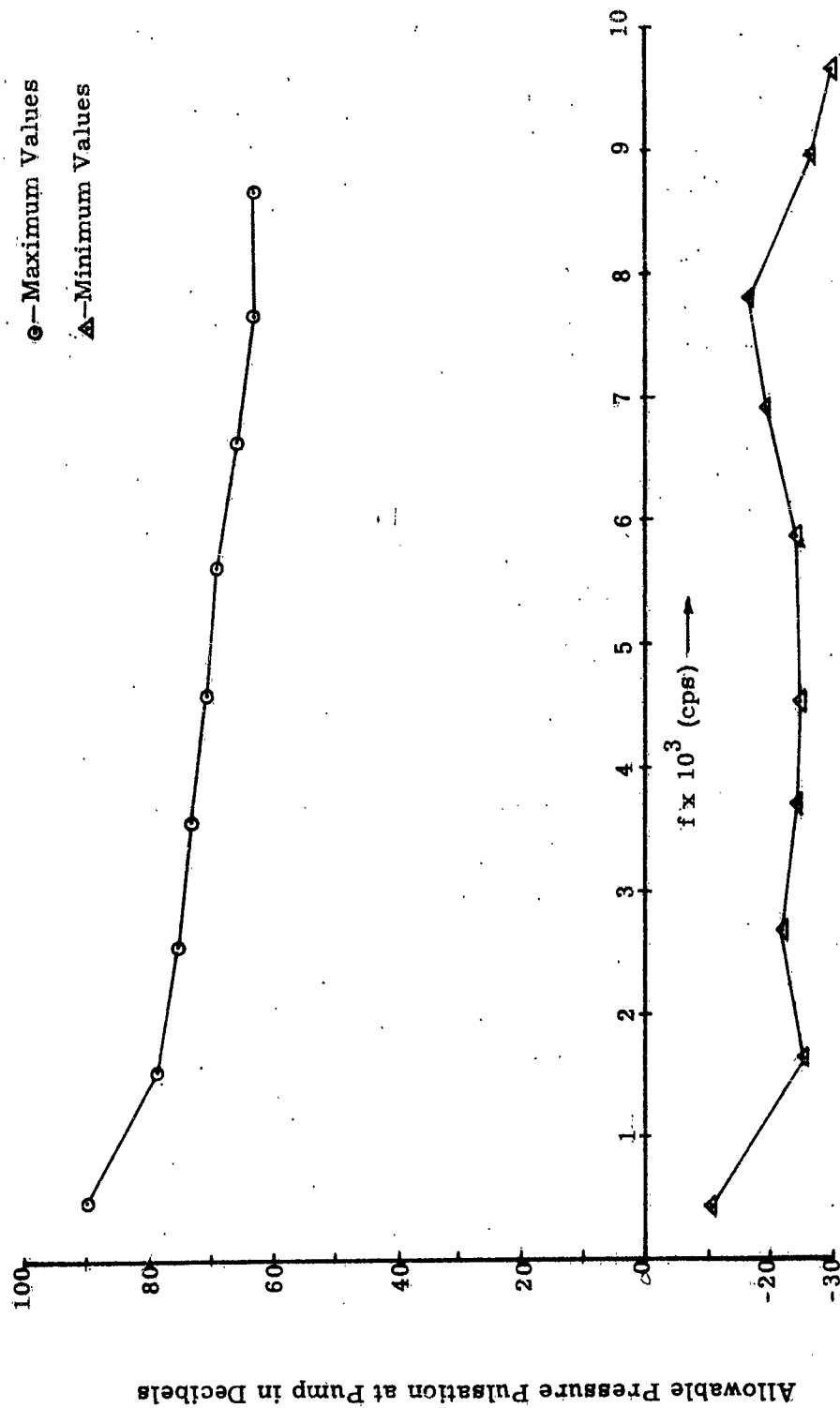Figure 15. Typical Segment of Allowable Pressure Spectrum at Pump, Example 3.

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Figure 16. Maximum and Minimum Values of the Allowable Pressure Spectrum at Pump, Example 3.

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